Rates of Turbulent Transfer to a Pipe Wall in the Mass Transfer Entry Region

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When a fluid in fully developed turbulent flow enters a section of pipe where mass (or heat) is exchanged between the fluid and the wall, the concentration profile in the fluid undergoes a rapid change. An originally flat profile changes to one exhibiting large gradients of concentration near the wall. The shape of the profile in the fluid leaving a transfer section will depend on the length of the transfer section until, for sufficiently long sections, a fully developed condition is reached. As a consequence of the development of the concentration profile, the local value of the transfer coefficient decreases from a very large value at the inlet to a minimum value downstream. If the transfer section is long enough, the minimum value will be the fully developed one.

The length of transfer section required to attain the fully developed condition is called the *entry length* for the exchange process. Most measurements of heat and mass transfer rates have been made in sections of length such that the variation of the transfer coefficient in the entry region, does not affect the results. Measurements of heat transfer rates for short transfer sections have been reported by Knudsen and Fargui (*7*), Johnk (*6*), and Aladyev (*1*). Such studies are complicated by the problem of thermally isolating a small transfer section from the remainder of the pipe; in this respect, mass transfer experiments are superior for entry region investigations. Meyerink and Friedlander (*10*) have measured the rates of dissolution of soluble sections of a pipe wall as small as 0.87 diam. in length. Reiss (*11*) obtained mass transfer data in the entry section using a diffusion-controlled electrode in the form of a ring mounted flush with the pipe wall. The electrolytic technique has an obvious advantage over the use of a soluble wall in that there are no changes in the wall geometry during the process.

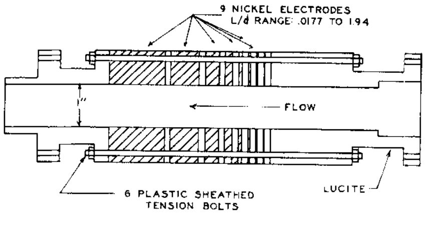

Fig. 1. Test section 1.

This paper reports on experiments performed to determine the effect of the length of the mass transfer section on the average rate of transfer. The electrolytic technique described by Eisenberg, Tobias, and Wilke (*3*), by Reiss (*11*), and by Reiss and Hanratty (*12*) was used. Ten mass transfer sections, with lengths of from 0.0177 to 4.31 diam., were employed. The Reynolds number was varied from 1,000 to 75,000, and the Schmidt number was constant at about 2,400. These measurements comprise a more comprehensive examination of the mass transfer entry region than has been previously reported.

CORRELATION OF THE MEASUREMENTS

Since the measurements are for the condition of large Schmidt numbers, the region in which there is a measurable concentration gradient is thinner than the viscous sublayer. Therefore, the curvature of the wall may be neglected, and the concentration field may be described in a cartesian coordinate system in which the x-axis is in the direction of flow, the y-axis extends inward from the pipe wall, and the z-axis is measured around the pipe circumference. Further, the distribution of average velocity through the concentration-boundary layer may be taken as a linear function of y:

$$U = \frac{u^{*2}}{\nu} y = \frac{f}{2\nu} U_b^2 y \qquad (1)$$

With Equation (1), the time-average equation for conservation of a scalar quantity in a turbulent field (*4*) for the system studied in this research is

$$\frac{u^{*2}}{\nu} y \frac{\partial \overline{C}}{\partial x} = \frac{\partial}{\partial x}\left(D \frac{\partial \overline{C}}{\partial x}\right) + \frac{\partial}{\partial y}\left(D \frac{\partial \overline{C}}{\partial y}\right) - \frac{\partial (\overline{uc})}{\partial x} - \frac{\partial (\overline{vc})}{\partial y} \qquad (2)$$

Linton and Sherwood (*8*) have proposed that the following simplified form of Equation (2) may provide a suitable description of the mass transfer process for very short transfer sections and large Schmidt numbers:

$$\frac{u^{*2}}{\nu} y \frac{\partial \overline{C}}{\partial x} = D \frac{\partial^2 \overline{C}}{\partial y^2} \qquad (3)$$

The neglection of $\dfrac{\partial (\overline{vc})}{\partial y}$ is probably valid; however, it is difficult to assess the importance of $\dfrac{\partial (\overline{uc})}{\partial x}$ and $\dfrac{\partial}{\partial x}\left(D \dfrac{\partial \overline{c}}{\partial x}\right)$. Since the turbulent-velocity component normal to the wall, v, varies at least as y^2 in the immediate vicinity of the wall, there is a region in which $\overline{vc}$ is small compared to $D \dfrac{\partial \overline{C}}{\partial y}$, and for sufficiently short transfer sections the entire concentration-boundary layer will be included in this region. Measurements of Reiss and Hanratty (*12*) indicate that the turbulent-velocity component in the direction of flow, u, varies as y near the wall; therefore, the type of argument given above cannot be used to neglect $\dfrac{\partial (\overline{uc})}{\partial x}$ for short transfer sections unless the absolute magnitudes of u and c are small compared to the magnitudes of U and $\overline{C}$.

Table 1. Physical Properties of the Electrolyte Solution of 25°C.

Density, g./cc.: 1.075
Viscosity, centipoise: 1.33
NaOH concentration, moles/liter: approximately 1.9
Diffusion coefficient for ferricyanide ion, sq. cm./sec.:
  Experimental: 5.15×10^{-6}
  Reference 3: 5.62×10^{-6}
Schmidt number (experimental): 2,400

The data in this paper has been deposited in tabular form as document 7530 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or for 35-mm. microfilm.

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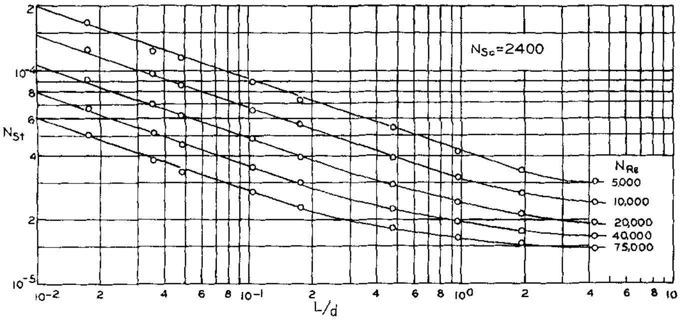

Fig. 2. Stanton number vs. L/d, N_{Re} parameters.

Integration of Equation (3) yields

$$\left\langle \frac{K}{u^*} \right\rangle = 0.810\, N_{Sc}^{-2/3}\, L^{+\ -1/3} \quad (4)$$

If the Blasius equation for the friction factor is used, Equation (4) may be written as

$$\langle N_{St} \rangle = 0.276\, N_{Re}^{-0.42}\, N_{Sc}^{-2/3} \left(\frac{L}{d}\right)^{-1/3} \quad (5)$$

Equation (4) is also valid for short transfer sections with laminar flow and leads to a form of the familiar Leveque equation:

$$\langle N_{St} \rangle = 1.615\, N_{Re}^{-2/3}\, N_{Sc}^{-2/3} \left(\frac{L}{d}\right)^{-1/3} \quad (6)$$

If an eddy diffusion coefficient is defined as

$$-\overline{vc} = \epsilon \frac{\partial \bar{c}}{\partial y} \quad (7)$$

and if $\frac{\partial \overline{uc}}{\partial x}$ is neglected, then the following dimensionless form of the mass balance can be written to describe the entire mass transfer section:

$$u^+ \frac{\partial \bar{c}}{\partial x^+} = \frac{\partial}{\partial y^+}\left[\left(N_{Sc} + \frac{\epsilon}{\nu}\right)\frac{\partial \bar{c}}{\partial y^+}\right] + \frac{\partial}{\partial x^+}\left[N_{Sc}\frac{\partial \bar{c}}{\partial x^+}\right] \quad (8)$$

If it can be assumed that N_{Sc} is constant and that u^+ and ϵ/ν are functions only of y^+ and x^+ and are not directly dependent on the pipe diameter, then $\bar{c}$ is a function of y^+, x^+, and N_{Sc}. The following functional relationship results:

$$\left\langle \frac{K}{u^*} \right\rangle = f(L^+, N_{Sc}) \quad (9)$$

It is to be noted that because of the assumption that the concentration field is not directly dependent on the pipe diameter, the Reynolds number does not appear as a parameter in Equation (9). This type of correlation for transfer data was suggested by Spalding (*13*) and by Gardner and Kestin (*4*).

Deissler (2) and Metzner and Friend (9) have developed relations between ϵ/ν and y^+ and used them to solve Equation (8). For high N_{Sc} and for the fully developed condition $\left(\frac{\partial \bar{c}}{\partial x^+} = \text{constant}\right)$, Deissler's analysis leads to

$$\frac{K}{u^*} = 0.112\, N_{Sc}^{-3/4} \quad (10)$$

and Metzner and Friend's, to

$$\frac{K}{u^*} = 0.085\, N_{Sc}^{-2/3} \quad (11)$$

Colburn's empirical relation suggests a slight dependence on Reynolds number:

$$\frac{K}{u^*} = 0.1155\, N_{Re}^{-0.075}\, N_{Sc}^{-2/3} \quad (12)$$

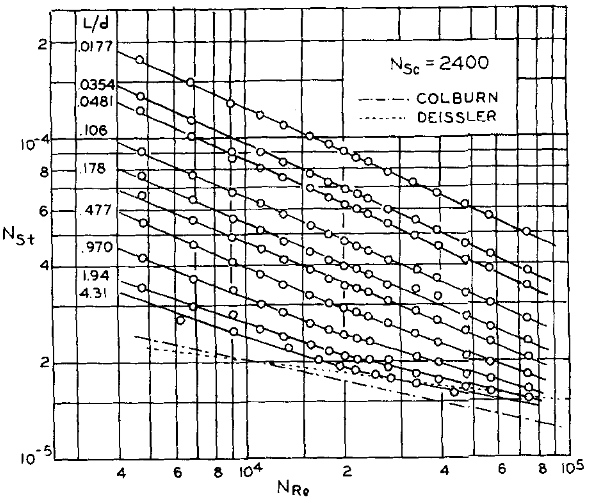

Fig. 3. Stanton number vs. Reynolds number L/d parameters.

EXPERIMENTAL

The experiments were performed in a flow loop of plastic pipe containing a test section in which short sections of the pipe wall were nickel electrodes. Downstream of the test section was a reference electrode of nickel pipe of much larger surface area than any of the test elements. A length of about 180 diam. of straight pipe preceded the test section. An electrolyte solution which was approximately 2 molar in sodium hydroxide and contained small equimolar quantities (approximately 0.01 to 0.0001 molar) of ferricyanide and ferrocyanide ions was circulated through the loop. Two interchangeable test sections were employed. The first, shown in Figure 1, had nine separate electrodes ranging in length from 0.0179 to 1.96 in. separated by ¼ in. thick Lucite disks and held together by six tension bolts. The metal and plastic disks were bored individually to a 1 in. I.D. before the test section was assembled. The inside of the completed section was reamed to a final inside diameter of 1.013 in. and polished smooth. The second test section was an electrode consisting of a 4.31 in. length of 1 in. I.D. nickel pipe. Both test sections had flanged socket-type of entries of Lucite construction. Discontinuities at the entry were removed by sanding and polishing.

The electrode reaction carried out on the surface of the test electrode was the reduction of ferricyanide:

$$\mathrm{Fe(CN)_6}^{-3} + e \rightarrow \mathrm{Fe(CN)_6}^{-4} \quad (13)$$

The reverse reaction occurred on the larger downstream electrode so that the bulk concentration of ferricyanide ions remained constant, even though the electrolyte was recycled.

Measurements were made in the region of the current-voltage curves where $\partial i/\partial E = 0$. In this region the reaction rate is limited by the rate at which ferricyanide ions diffuse to the electrode, and the surface concentration is zero.

The rate of mass transfer due to diffusion is given by

$$N = \frac{i/A}{nF}(1 - T) \quad (14)$$

where T is the transference number of the species transferred. When $T << 1$ the effect of migration may be neglected. For the conditions reported in this study, T for the ferricyanide ion had a maximum value of 0.002, and migration effects were therefore considered negligible.

Diffusion-limited currents for the various electrodes were obtained for Reynolds numbers between 1,000 and 75,000. The measurements reported were made with ferricyanide ion concentrations of approximately 0.001

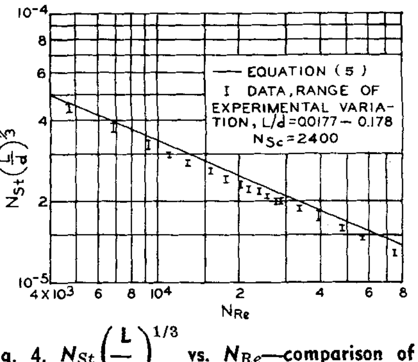

Fig. 4. $N_{St}\left(\frac{L}{d}\right)^{1/3}$ vs. N_{Re}—comparison of Equation (5) to data.

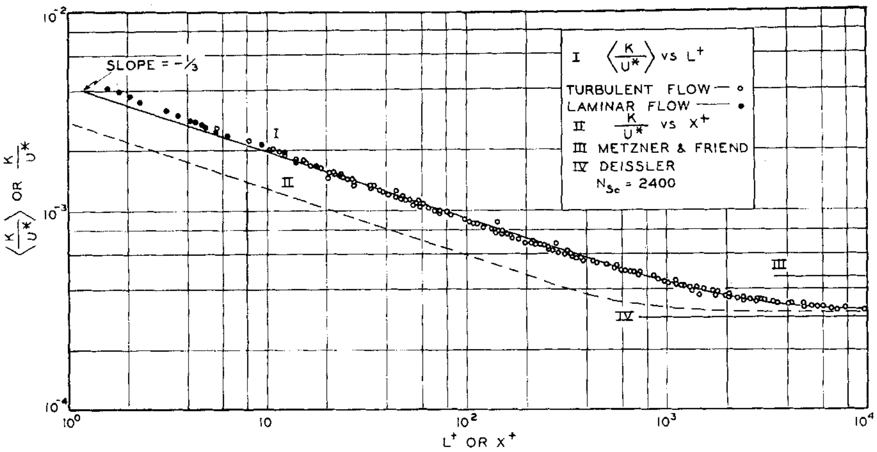

Fig. 5. Correlation of mass transfer data using Equation (9).

molar. Other experiments in this laboratory have shown that the measured Stanton number is independent of the ferricyanide ion concentration over the range of 0.001 to 0.01 molar. Mass transfer coefficients were calculated from the following equation with $n = 1$ and $C_w = 0$:

$$K = \frac{i/A}{nF(C_b - C_w)} \qquad (15)$$

Measurements of N_{St} for laminar flow with electrode lengths up to 1.94 diameters were found to be proportional to $(L/d)^{-1/3}$ and $N_{Re}^{-2/3}$. These relations have been obtained for Reynold numbers from 100 to 2,200. The laminar-flow data were used with Equation (6) to evaluate the Schmidt number for the solution. The value thus determined is in good agreement with that reported by Eisenberg (3) as may be seen in Table 1.

RESULTS

Measured Stanton numbers are plotted as a function of transfer section length in Figure 2. For short transfer sections the Stanton number is proportional to $(L/d)^{-1/3}$ as predicted by Equation (5). The range of L/d over which this dependency holds decreases as the Reynolds number is increased. For high Reynolds numbers the Stanton numbers for electrodes with $L/d \geqq 1.94$ approach the fully developed values predicted by Equation (10). The Stanton number is plotted as a function of N_{Re} with L/d as a parameter in Figure 3. The data suggest that the Stanton numbers for all the transfer lengths vary as $N_{Re}^{-0.42}$ at sufficiently low flow rates. Although the fully developed condition was approached in a few of the runs, none of the transfer sections studied were long enough for the entry effect to be entirely negligible.

The data for L/d ratios from 0.0177 to 0.179 are compared with the limiting solution represented by Equation (5) in Figure 4. Equation (5) correctly describes the dependency of N_{St} on L/d and N_{Re} but predicts values about 7% high. The reason for this discrepancy is not known.

The mass transfer coefficients are plotted in the form suggested by Equation (9) in Figure 5. This type of correlation satisfactorily represents all of the measurements reported in this paper. It appears that the assumption that ϵ/ν is a function only of y^+ and x^+ is valid at high Schmidt numbers. Equation (9) might not be as accurate for lower Schmidt numbers when the concentration-boundary layer becomes thicker (6). Figure 5 also shows a curve representing local values of K/u^* as a function of x^+. These were obtained from the original data by computing average transfer coefficients over small increments of L^+. The agreement with Equation (10) is clearly indicated. The discrepancy of the data at small values of L^+ and the limiting solution shown in Figure 4 is again evident in Figure 5. The fact that there is an overlap between the data for laminar and turbulent flow would seem to indicate that the neglect of $\frac{\partial(\overline{uc})}{\partial x}$ is not the cause of the discrepancy.

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NOTATION

A = electrode area
C = concentration of diffusing species
c = fluctuating component of concentration
C_b = bulk concentration of diffusing species
C_w = concentration at the tube wall
d = pipe diameter
D = molecular diffusion coefficient
E = applied voltage
f = friction factor
F = Faraday's constant
i = electrode current
K = time average mass transfer coefficient
L = electrode or transfer section length
L^+ = dimensionless length, $L^+ = \frac{Lu^*}{\nu}$
n = number of electrons involved in electrode reaction
U = time average local velocity parallel to pipe axis
u^* = friction velocity
U_b = bulk average velocity
u,v,w = fluctuating velocity components
x,y,z = cartesian coordinate axes
x^+,y^+ = dimensionless cartesian coordinates, $x^+ = \frac{xu^*}{\nu}$ $y^+ = \frac{yu^+}{\nu}$
ν = kinematic viscosity
ϵ = eddy diffusion coefficient
N_{Re} = Reynold's number, dU_b/ν
N_{Sc} = Schmidt number, ν/D
N_{St} = Stanton number, K/U_b
— = (superscript bar) indicates time average
<> = indicates average over range $x = 0$ to $x = L$

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